

CHAPTER 4

SPRINGS

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4.1 INTRODUCTION

Mechanical springs are used in machine designs to exert force, provide flexibility, and to store or absorb energy. Springs are manufactured for many different applications such as compression, extension, torsion, power, and constant force. Depending on the application, a spring may be in a static, cyclic or dynamic operating mode. A spring is usually considered to be static if a change in deflection or load

occurs only a few times, such as less than 10,000 cycles during the expected life of the spring. A static spring may remain loaded for very long periods of time. Cyclic springs are flexed repeatedly and can be expected to exhibit a higher failure rate due to fatigue. Dynamic loading refers to those intermittent occurrences of a load surge such as a shock absorber inducing higher than normal stresses on the spring.

The reliability of a spring will depend not only on the material and design characteristics, but to a great extent on the operating environment. Most springs are made of steel and therefore corrosion protection has a significant impact on reliability. Material properties, the processes used in the manufacturing of the spring, operating temperature, and corrosive media must all be known before any estimate of spring reliability can be made.

4.2 FAILURE MODES

The operating life of a mechanical spring arrangement is dependent upon the susceptibility of the materials to corrosion and stress levels (static, cyclic or dynamic). The most common failure modes for springs are fracture due to fatigue and excessive loss of load due to stress relaxation. Table 4-1 is a list of failure mechanisms and causes of spring failure. Other failure mechanisms and causes may be identified for a specific application to assure that all considerations of reliability are included in the prediction. Typical failure rate considerations include: level of loading, operating temperature, cycling rate and corrosive environment.

Table 4-1. Failure Modes for a Mechanical Spring

APPLICATION	FAILURE MODES	FAILURE CAUSES
- Static (constant deflection or constant load)	- Load loss - Creep - Set	- Parameter change - Hydrogen embrittlement
- Cyclic (unidirectional or reverse stress, 10,000 cycles or more during the life of the spring)	- Fracture	- Material flaws - Hydrogen embrittlement - Stress concentration due to tooling marks and rough finishes - Corrosion - Misalignment
- Dynamic	- Fracture	- Maximum load ratio exceeded

In many applications, compression and extension springs are subjected to elevated temperatures at high stresses which can result in relaxation or loss of load. This condition is often referred to as "set". After the operating conditions are determined, set can be predicted and allowances made in the spring design. When no set is allowed in the application, the spring manufacturer may be able to preset the spring at temperatures and stresses higher than those to be encountered in the operating environment.

Most extension spring failures occur in the area of the spring end. For maximum reliability, the spring wire must be smooth with a gradual flow into the end without tool marks or other stress risers. The spring ends should be made as an integral part of the coil winding operation and the bend radius should be at least one and one-half times the wire diameter.

The S_{10} value for a spring is the number of cycles that 90% of the springs operating at the published stress level can be expected to complete or exceed before exhibiting the first evidence of fatigue. If an S_{10} value for the spring can be obtained, this value should be used in conjunction with the environmental multiplying factors contained in this Chapter. The procedure for estimating spring failure rates contained herein is intended to be used in the absence of specific S_{10} data.

4.3 FAILURE RATE CONSIDERATIONS

The following paragraphs describe the terms and parameters used in developing failure rate equations for springs.

4.3.1 Static Springs

Static springs can be used in constant deflection or constant load applications. A constant deflection spring is cycled through a specified deflection range, the loads on the spring causing some set or relaxation which in turn lowers the applied stress. The spring may relax with time and reduce the applied load. Under constant load conditions, the load applied to the spring does not change during operation. Constant load springs may set or creep, but the applied stress is constant. The constant stress may result in fatigue lives shorter than those found in constant deflection applications.

4.3.2 Cyclic Springs

Cyclic springs can be classified as being unidirectional or reverse loaded. In one case, the stress is always applied in the same direction, while in the other, stress is applied first in one direction then in the opposite direction. [Figure 4.18](#) shows the relationship between the cycle rate of a spring and its effect on failure rate.

4.3.3 Modulus of Rigidity

The modulus of rigidity (G_M) is a material property defining the resistance to shearing stresses for the spring material, the ratio of shearing stress to shear strain. Typical values are provided in [Table 4-2](#).

4.3.4 Modulus of Elasticity

The modulus of elasticity provides a measure of elasticity in tension for the spring material. Typical values are provided in [Table 4-2](#).

4.3.5 Spring Index

Spring index (r) is the ratio of mean coil diameter to wire diameter. A spring with a high index will tend to tangle or buckle.

4.3.6 Spring Rate

Spring rate (R) is the change in load per unit deflection, a measure of spring relaxation.

4.3.7 Shaped Springs

If the spring has a variable diameter such as occurs for conical, barrel and hourglass springs, the spring can be divided analytically into smaller increments and the failure rate calculated for each. The failure rate for the total spring is computed by adding the rates for the increments.

4.3.8 Number of Active Coils

For compression springs with closed ends, either ground or not ground, the number of active coils is two less than the total number of coils. There is some activity in the end coils, but during deflection, some active material comes in contact with the end coils and becomes inactive. Therefore, the total number of coils minus two is a good approximation for the number of active coils. For extension springs, all coils are active.

4.3.9 Tensile Strength

The tensile strength provides a measure of spring material deformation or set as a function of stress. Values of tensile strength are included in [Table 4-3](#).

4.3.10 Corrosive Environment

Corrosion will reduce the load-carrying capability of a spring and its life. The precise effect of a corrosive environment on spring performance is difficult to predict. The reliability of a spring in terms of fatigue life and load-carrying ability will be affected

by corrosion, the quantitative effect being very hard to predict. Springs are almost always in contact with other metal parts. If a spring is to be subjected to a corrosive environment, the use of inert materials provides the best defense against corrosion. Protective coatings can also be applied. In special situations, shot peening can be used to prevent stress corrosion and cathodic protection systems can be used to prevent general corrosion. The spring material is normally more noble (chemically resistant to corrosion) than the structural components in contact with it because the lesser noble alloy will be attacked by the electrolyte. The effects of corrosion on spring reliability must be based on experience data considering the extent of a corrosive environment. If corrosive protection is known to be applied to the spring during the manufacturing process, a multiplying factor, C_R , of 1.0 is used in conjunction with the base failure rate. Values of C_R greater than 1.0 are used based on the user's experience with the spring and the operating environment.

4.3.11 Manufacturing Processes

The following effects of manufacturing processes need to be considered in evaluating a design for reliability:

- Sharp corners and similar stress risers should be minimized.
- The hardness of the spring material can be sensitive to plating and baking operations. Quality control procedures for these operations should be reviewed. A multiplying factor, C_M , of 1.0 should be used in conjunction with the base failure rate for known acceptable quality control procedures; otherwise a higher value for the multiplying factor should be used based on previous experience with the manufacturer.

4.3.12 Other Reliability Considerations for Springs

The most common failure modes of springs include fracture due to fatigue and excessive loss of load. A reliability analysis should include a review of the following items to assure maximum possible life:

- When a spring is loaded or unloaded, a surge wave may transmit torsional stress to the point of restraint. The impact velocity should be determined to assure that the maximum load rating of the spring is not exceeded.
- Operating temperature should be determined. Both high and low temperature conditions may require consideration of specialized materials.
- Exposure to electrical fields may magnetize the spring material and cause fatigue failure.

4.4 FAILURE RATE MODELS

4.4.1 Compression Spring

The compression spring is the most commonly used spring in machine designs. An example of a compression spring is shown in Figure 4.1.

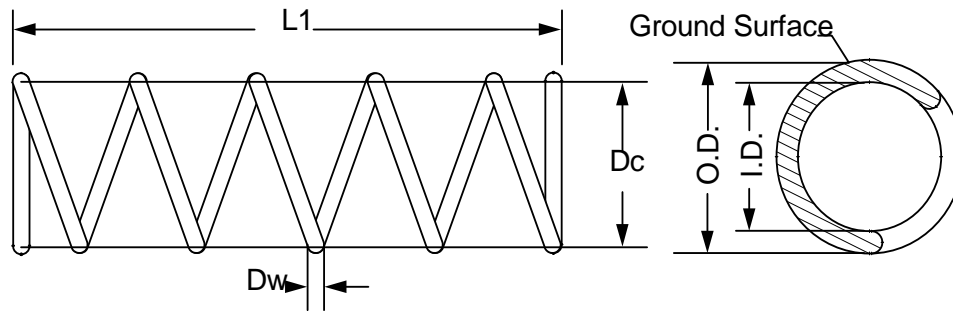


Figure 4.1 Typical Helical Compression Spring

The failure rate of a compression spring depends upon the stress on the spring and the relaxation provided by the material. This relaxation (change in load per unit deflection) is referred to as the spring rate, R . The spring rate for a compression spring is calculated using Equation (4-1).

$$R = \frac{G_M (D_W)^4}{8 (D_C)^3 N_a} = \frac{P_L}{L_1 - L_2} \quad (4-1)$$

- Where:
- R = Spring rate, lbs/in
 - G_M = Modulus of rigidity, lbs/in²
 - D_W = Wire diameter, in
 - D_c = Mean diameter of spring, in
 - N_a = Number of active coils (See [Section 4.3.8](#))
 - P_L = Load, lbs
 - L_1 = Initial length of spring, in
 - L_2 = Final deflection of spring, in

The spring rate can be determined experimentally by deflecting the spring to 20% of available deflection and measuring the load (P_1) and spring length (L_1). Next, the spring is deflected to 80% of available deflection measuring the load (P_2) and spring length (L_2), being certain that no coils other than the closed ends are touching. The spring rate is then calculated as follows:

$$R = \frac{P_2 - P_1}{L_1 - L_2} = \frac{P_L}{L_1 - L_2} \quad (4-2)$$

Stress in the spring is also proportional to the load, P_L according to the following relationship:

$$S_G = \frac{8 P_L D_C}{\pi D_W^3} K_W \quad (4-3)$$

Where: S_G = Spring stress, lbs/in²
 K_W = Spring concentration factor (See equation 4-4)
 D_C = Mean coil diameter, in
 D_W = Wire Diameter, in

The spring concentration factor, K_W is a function of the Spring index (ratio of the coil diameter to wire diameter).

$$K_W = \frac{4r - 1}{4r - 4} + \frac{0.615}{r} \quad (4-4)$$

Where: r = Spring index = D_C / D_W

P_L in Equation (4-1) can be substituted into Equation (4-3) for a stress level equation, and the spring failure rate can be determined from a ratio of stress level to the material tensile strength according to the following empirically derived relationship ([Reference 14](#)):

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S_G}{T_S} \right)^3 = \lambda_{SP,B} \left(\frac{8 P_L D_C K_W}{\pi T_S D_W^3} \right)^3 \quad (4-5)$$

Where:

$$\lambda_{SP} = \text{Failure rate of spring, failures/million hours}$$

$$\lambda_{SP,B} = \text{Base failure rate for spring, 23.8 failures/million hours}$$

$$T_S = \text{Material tensile strength, lbs/in}^2$$

A generalized equation that adjusts the base failure rate of a compression spring considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_G \cdot C_{DW} \cdot C_{DC} \cdot C_N \cdot C_Y \cdot C_L \cdot C_K \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-6)$$

Where:

- C_G = Multiplying factor which considers the effect of the material rigidity modulus on the base failure rate (See [Table 4-2](#))
- C_{DW} = Multiplying factor which considers the effect of the wire diameter on the base failure rate (See [Figure 4.8](#))
- C_{DC} = Multiplying factor which considers the effect of coil diameter on the base failure rate (See [Figure 4.9](#))
- C_N = Multiplying factor which considers the effect of the number of active coils on the base failure rate (See [Figure 4.10](#))
- C_Y = Multiplying factor which considers the effect of material tensile strength, T_s , on the base failure rate (See [Table 4-3](#))
- C_L = Multiplying factor which considers the effect of spring deflection on the base failure rate (See [Figure 4.11](#))
- C_K = Multiplying factor which includes the spring concentration factor on the base failure rate (See [Figure 4.12](#))
- C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))
- C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
- C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

The parameters in the failure rate equation can be located on an engineering drawing by knowledge of design standards or by actual measurements. Other manufacturing, quality, and maintenance contributions to failure rate are included in the base failure rate as determined from field performance data.

4.4.2 Extension Spring

Helical extension springs store energy in spring tensioning devices and are used to exert a pulling force. Most helical extension springs are coiled with initial tension, equal to the minimum force required to separate adjacent coils. Extension springs require a method of attachment to other parts of the assembly. For extension springs, all coils are active and N_a will be equal to the number of coils. Otherwise, the failure rate equations for extension springs are similar to compression springs and the procedures in [Section 4.4.1](#) should be used.

4.4.3 Torsion Spring

Helical torsion springs are used to apply a torque or store rotational energy, the most common application, the clothes pin. Torsion springs are stressed in bending as shown in Figure 4.2. A torsion spring should always be loaded in a direction that causes its body diameter to decrease because of increased stresses when the spring is loaded in a direction which increases body diameter.

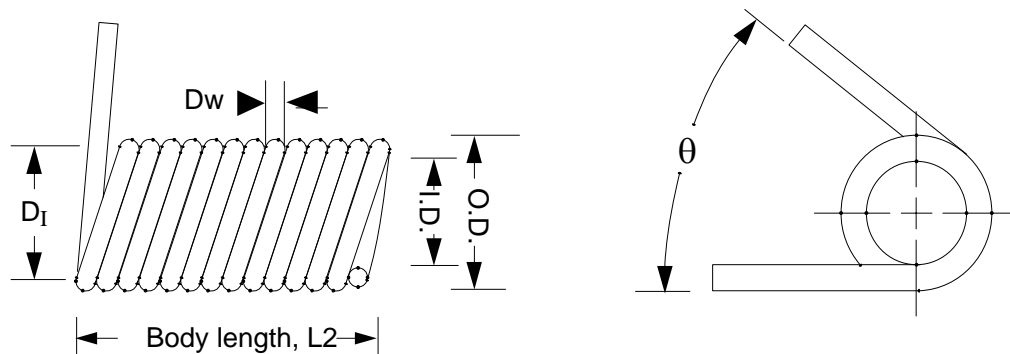


Figure 4.2 Typical Helical Torsion Spring

The mean diameter of a helical torsion spring is equal to:

$$D_I = \frac{ID + OD}{2} \quad (4--7)$$

The spring diameter will change with deflection according to the following equation:

$$D_c = \frac{D_I N_a}{N_a + \theta} \quad (4-8)$$

Where: D_C = Mean diameter after deflection
 D_I = Initial mean diameter, in.
 θ = Angular deflection from free position, revolutions
 N_a = Number of active coils

Most torsion springs are close-wound, with body length equal to wire diameter multiplied by the number of turns plus one. When the spring is deflected in a direction which reduces its coil diameter, body length increases to L_2 according to the following equation:

$$L_2 = D_W (N_a + 1 + \theta) \quad (4-9)$$

Where: D_W = Wire diameter, in

Stress in torsion springs is due to bending and for round wire is calculated with the following equation:

$$S = \frac{3 E_M D_W \theta}{\pi D_I N_a} \quad (4-10)$$

Where: S = Bending stress, lbs/in²
 E_M = Modulus of Elasticity, lbs/in²
 D_W = Wire diameter, in
 θ = Angular deflection, revolutions
 D_I = Mean diameter of spring, in
 N_a = Number of active coils ([See Section 4.3.8](#))

The equation to determine the failure rate of a torsion spring can be written as follows:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11)$$

From this equation a generalized equation can be developed containing a base failure rate with applicable multiplying factors:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_{DW} \cdot C_N \cdot C_Y \cdot C_L \cdot C_{CS} \cdot C_{DC} \cdot C_R \cdot C_M \quad (4-12)$$

- Where:
- λ_{SP} = Failure rate of torsion spring, failures/million hours
 - $\lambda_{SP,B}$ = Base failure rate for torsion spring, 14.3 failures/million hours
 - C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))
 - C_{DW} = Multiplying factor which considers the effect of the wire diameter on the base failure rate (See [Figure 4.8](#))
 - C_N = Multiplying factor which considers the effect of the number of active coils on the base failure rate (See [Figure 4.10](#))
 - C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))
 - C_L = Multiplying factor which considers the effect of spring deflection on the base failure rate (See [Figure 4.13](#))
 - C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))
 - C_{DC} = Multiplying factor which considers the effect of coil diameter on the base failure rate (See [Figure 4.19](#))
 - C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
 - C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

4.4.4 Curved Washer

Curved washers are used to secure fasteners, distribute loads, absorb vibrations and axial end play, and other similar applications. A typical curved washer is shown in [Figure 4.3](#). A special type of curved washer, the Belleville washer, is discussed in

Section 4.4.6. When a load is applied to a curved washer it tends to flatten causing radial and circumferential strains. This elastic deformation constitutes the spring action. Stress is not distributed uniformly in curved washers, the greatest stress occurring at the convex inner edge. Curved washers exert a relatively light thrust load. Bearing surfaces should be hard to prevent washer corners from scoring the shaft.

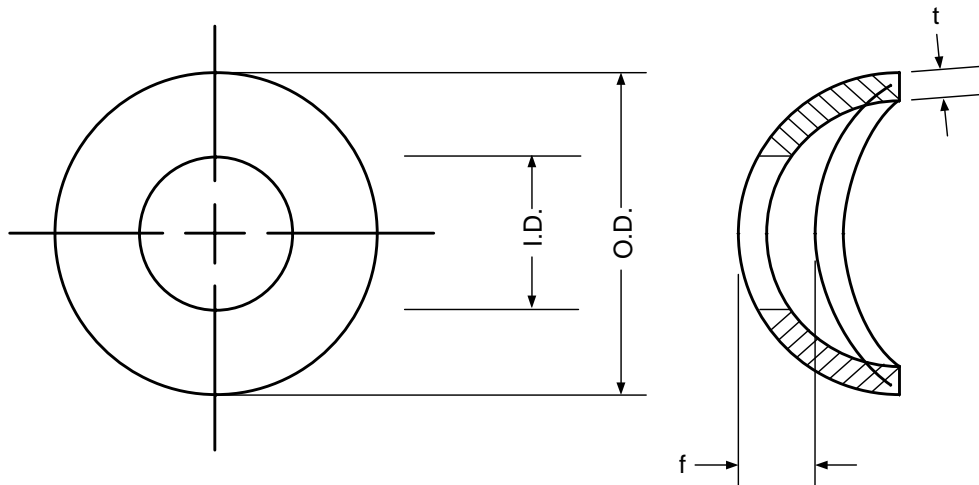


Figure 4.3 Typical Curved Washer

The stress on a curved washer is:

$$S = \frac{6 E_M f t}{(OD)^2} \quad (4-13)$$

Where:

- S = Bending stress, lb/in²
- E_M = Modulus of Elasticity, lb/in²
- f = Washer deflection, in
- t = Washer thickness, in
- OD = Outside Diameter, in

The failure rate of a curved washer is determined using the following equation:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11) \text{ ref}$$

A generalized equation that adjusts the base failure rate of a curved washer considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_t \cdot C_D \cdot C_Y \cdot C_f \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-14)$$

Where: λ_{SP} = Failure rate of curved washer, failures/million hours

$\lambda_{SP,B}$ = Base failure rate for curved washer, 1.1 failures/million hours

C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))

C_t = Multiplying factor which considers the effect of the material thickness on the base failure rate (See [Figure 4.14](#))

C_D = Multiplying factor which considers the effect of washer diameter on the base failure rate (See [Figure 4.15](#))

C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))

C_f = Multiplying factor which considers the effect of washer deflection on the base failure rate (See [Figure 4.16](#))

C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))

C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))

C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

4.4.5 Wave Washer

Wave washers are used to apply moderate thrust loads when radial space is limited. A typical wave washer is shown in [Figure 4.4](#).

The stress on a wave washer is given by:

$$S = \frac{0.3 \pi E_M f t N^2}{D^2} \quad (4-15)$$

Where: S = Bending stress, lbs/in²
 E_M = Modulus of Elasticity, lbs/in²
 f = Deflection, in
 t = Material thickness, in
 N = Number of waves
 D = Mean diameter, in = (OD + ID)/2
 OD = Outside Diameter, in
 ID = Inside Diameter, in

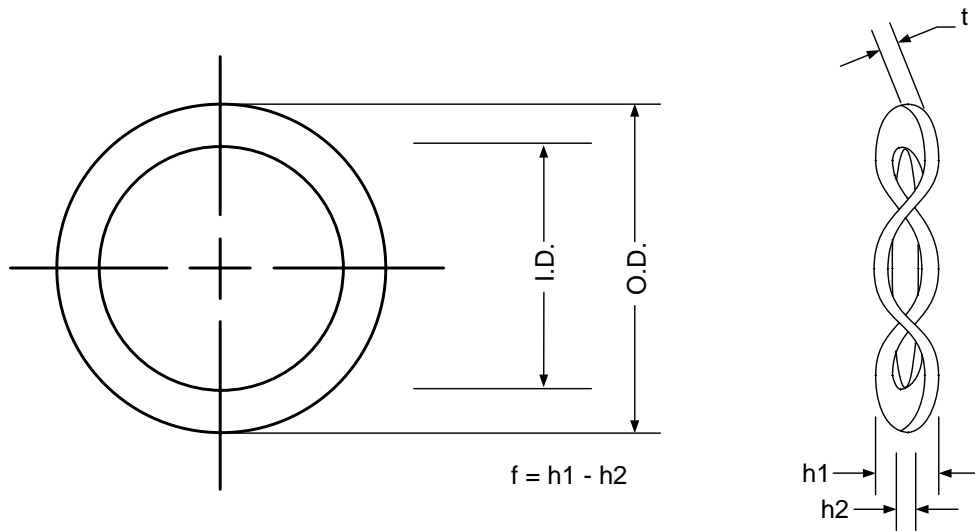


Figure 4.4 Typical Wave Washer

The failure rate of a wave washer is determined using the following equation:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11) \text{ ref}$$

A generalized equation that adjusts the base failure rate of a wave washer considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_t \cdot C_D \cdot C_Y \cdot C_f \cdot C_{NW} \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-16)$$

- Where:
- λ_{SP} = Failure rate of wave washer, failures/million hours
 - $\lambda_{SP,B}$ = Base failure rate for wave washer, 1.9 failures/million hours
 - C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))
 - C_t = Multiplying factor which considers the effect of the material thickness on the base failure rate (See [Figure 4.14](#))
 - C_D = Multiplying factor which considers the effect of washer diameter on the base failure rate (See [Figure 4.15](#))
 - C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))
 - C_f = Multiplying factor which considers the effect of washer deflection on the base failure rate (See [Figure 4.16](#))
 - C_{NW} = Multiplying Factor which considers the number of waves on the base failure rate (See [Table 4-4](#))
 - C_{CS} = Multiplying factor which considers the effect of cycle rate on the base failure rate (See [Figure 4.18](#))
 - C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
 - C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

4.4.6 Belleville Washer

When a load is applied to a Belleville washer it tends to flatten causing radial and circumferential strains. This elastic deformation creates the spring action. A typical Belleville washer is shown in Figure 4.5. Belleville washers are capable of providing

very high loads at small deflections. Stress is not distributed uniformly in Belleville washers. The highest stress occurs at the top inner edge and can be estimated with the following equation:

$$S = \frac{E_M f R}{1 - \mu^2} \cdot \left(\frac{t}{a^2} \right) \quad (4-17)$$

Where: S = Bending stress, lbs/in²
 E_M = Modulus of Elasticity, lbs/in²
 f = Deflection, in
 μ = Poisson's Ratio
 R = Dimension factor (See Figure 4.17)
 t = Material thickness, in
 a = O.D./2, in

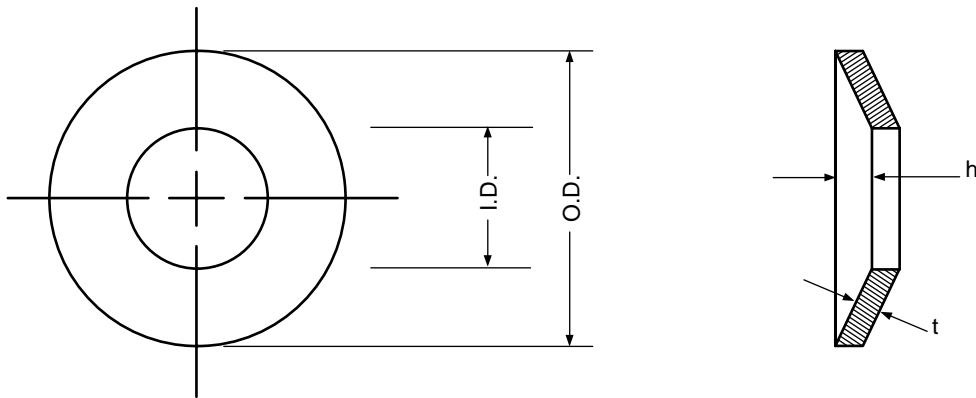


Figure 4.5 Typical Belleville Washer

The failure rate of a belleville washer is determined using the following equation:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11) \text{ ref}$$

A generalized equation that adjusts the base failure rate of a belleville washer considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_t \cdot C_D \cdot C_f \cdot C_Y \cdot C_S \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-18)$$

- Where:
- λ_{SP} = Failure rate of belleville washer, failures/million hours
 - $\lambda_{SP,B}$ = Base failure rate for belleville washer, 2.6 failures/million hours
 - C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))
 - C_t = Multiplying factor which considers the effect of material thickness on the base failure rate (See [Figure 4.14](#))
 - C_D = Multiplying factor which considers the effect of washer size on the base failure rate (See [Figure 4.15](#))
 - C_f = Multiplying factor which considers the effect of washer deflection under load on the base failure rate (See [Figure 4.16](#))
 - C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))
 - C_S = Multiplying factor for compressive stress (See [Figure 4.17](#))
 - C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))
 - C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
 - C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

4.4.7 Cantilever Spring

Cantilever springs are fabricated from flat strip material which stores and releases energy upon being deflected by an external load. A typical cantilever spring is shown in Figure 4.6. In complex designs, only a small part of the device may be functioning as a spring, and for analytical purposes, that portion which is active during operation may be considered as an independent device.

The bending stress for cantilever springs can be determined as follows:

$$S = \frac{3E_M f t}{2L^2} \quad (4-19)$$

Where: S = Bending stress, lbs/in²
 E_M = Modulus of elasticity, lbs/in²
 f = deflection, in
 t = thickness, in
 L = length, in

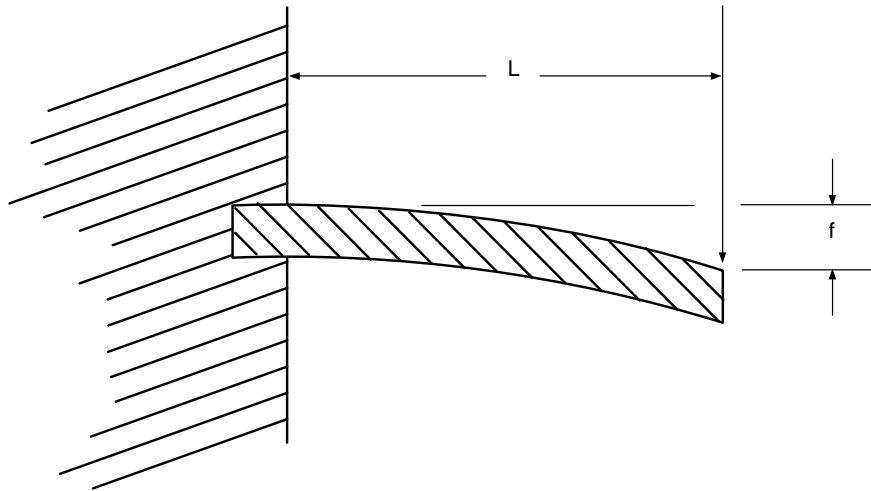


Figure 4.6 Typical Cantilever Spring

The failure rate of a cantilever spring is determined using the following equation:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11) \text{ ref}$$

A generalized equation that adjusts the base failure rate of a cantilever spring considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_t \cdot C_L \cdot C_f \cdot C_Y \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-20)$$

Where:

- λ_{SP} = Failure rate of cantilever spring, failures/million hours
- $\lambda_{SP,B}$ = Base failure rate for cantilever spring, 1.1 failures/million hours
- C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))
- C_t = Multiplying factor which considers the effect of material thickness on the base failure rate (See [Figure 4.14](#))
- C_L = Multiplying factor which considers the effect of spring length on the base failure rate (See [Figure 4.20](#))
- C_f = Multiplying factor which considers the effect of spring deflection on the base failure rate (See [Figure 4.16](#))
- C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))
- C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))
- C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
- C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))

4.4.8 Beam Spring

Beam springs are usually rectangular in shape and formed into an arc as shown in Figure 4.7. Assuming the ends are free to laterally expand, stress can be computed as follows:

$$S = \frac{6 E_M f t}{L^2} \quad (4-21)$$

Where:

- S = Bending stress, lbs/in²
- E_M = Modulus of elasticity, lb/in²
- f = Spring deflection, in
- t = Material thickness, in
- L = Active spring length, in

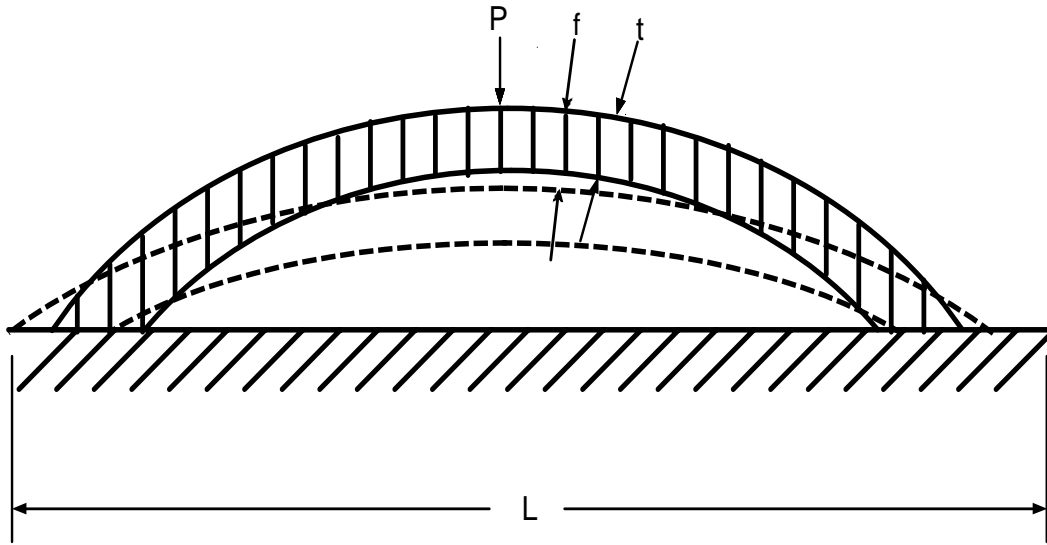


Figure 4.7 Typical Beam Spring

The failure rate of a beam spring is determined using the following equation:

$$\lambda_{SP} = \lambda_{SP,B} \left(\frac{S}{T_S} \right)^3 \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-11) \text{ ref}$$

A generalized equation that adjusts the base failure rate of a beam spring considering anticipated operating conditions can be established:

$$\lambda_{SP} = \lambda_{SP,B} \cdot C_E \cdot C_t \cdot C_L \cdot C_f \cdot C_Y \cdot C_{CS} \cdot C_R \cdot C_M \quad (4-22)$$

Where: λ_{SP} = Failure rate of beam spring, failures/million hours

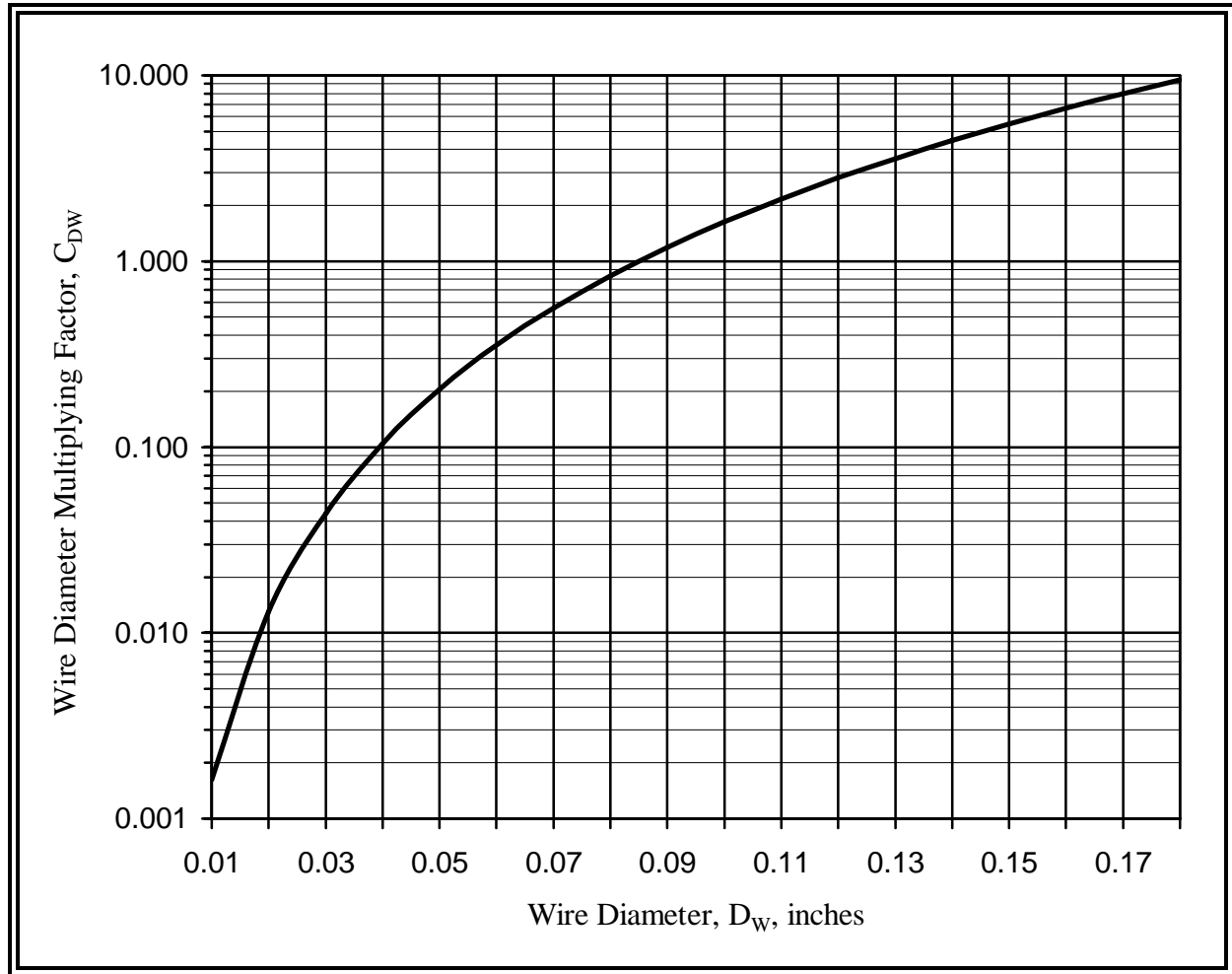
$\lambda_{SP,B}$ = Base failure rate for beam spring, 4.4 failures/million hours

C_E = Multiplying factor which considers the effect of the material elasticity modulus on the base failure rate (See [Table 4-2](#))

C_t = Multiplying factor which considers the effect of material thickness on the base failure rate (See [Figure 4.14](#))

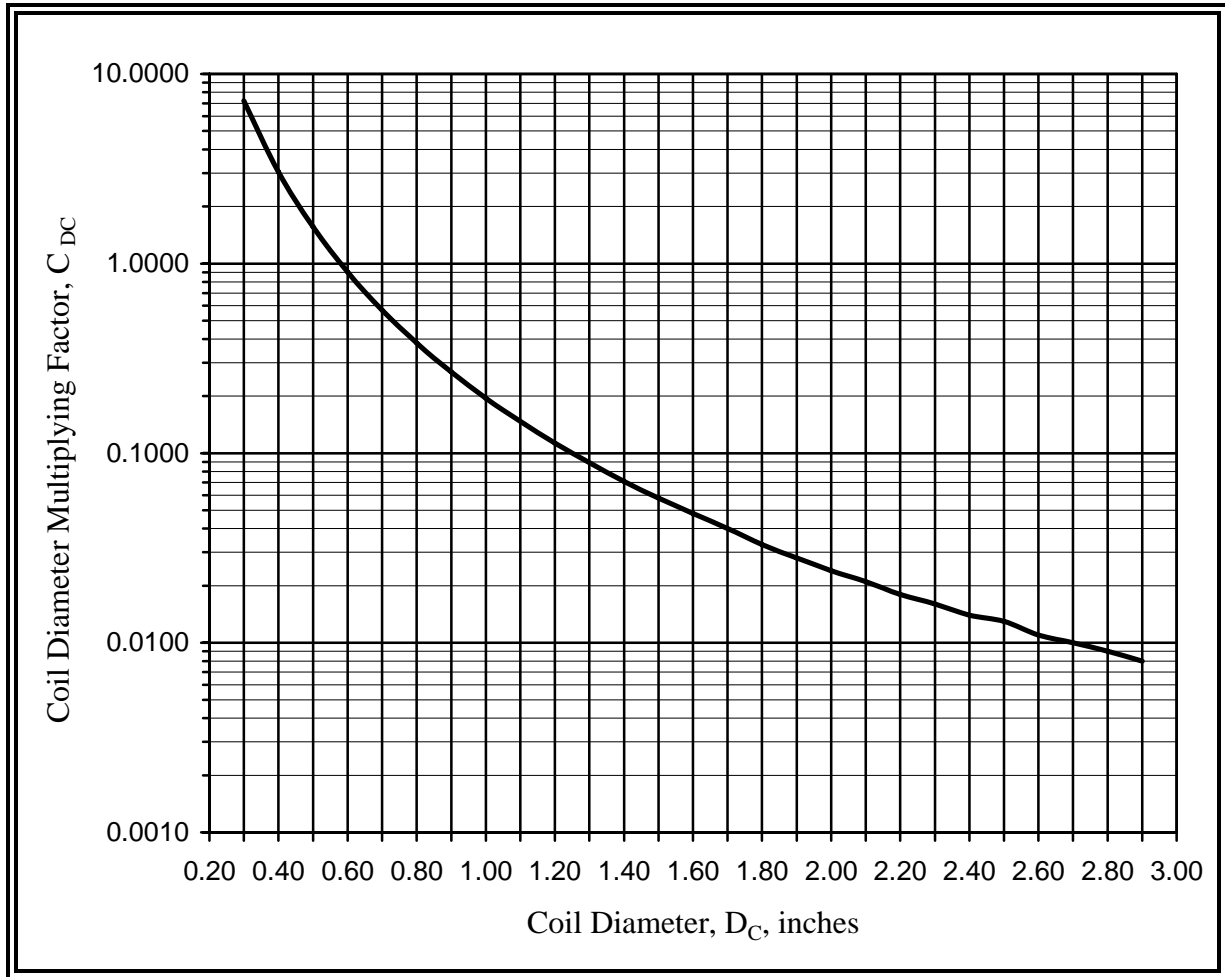
C_L = Multiplying factor which considers the effect of spring length on the base failure rate (See [Figure 4.20](#))

- C_f = Multiplying factor which considers the effect of spring deflection on the base failure rate (See [Figure 4.16](#))
- C_Y = Multiplying factor which considers the effect of material tensile strength on the base failure rate (See [Table 4-3](#))
- C_{CS} = Multiplying factor which considers the effect of spring cycle rate on the base failure rate (See [Figure 4.18](#))
- C_R = Multiplying factor which considers the effect of a corrosive environment on the base failure rate (See [Section 4.3.10](#))
- C_M = Multiplying factor which considers the effect of the manufacturing process on the base failure rate (See [Section 4.3.11](#))



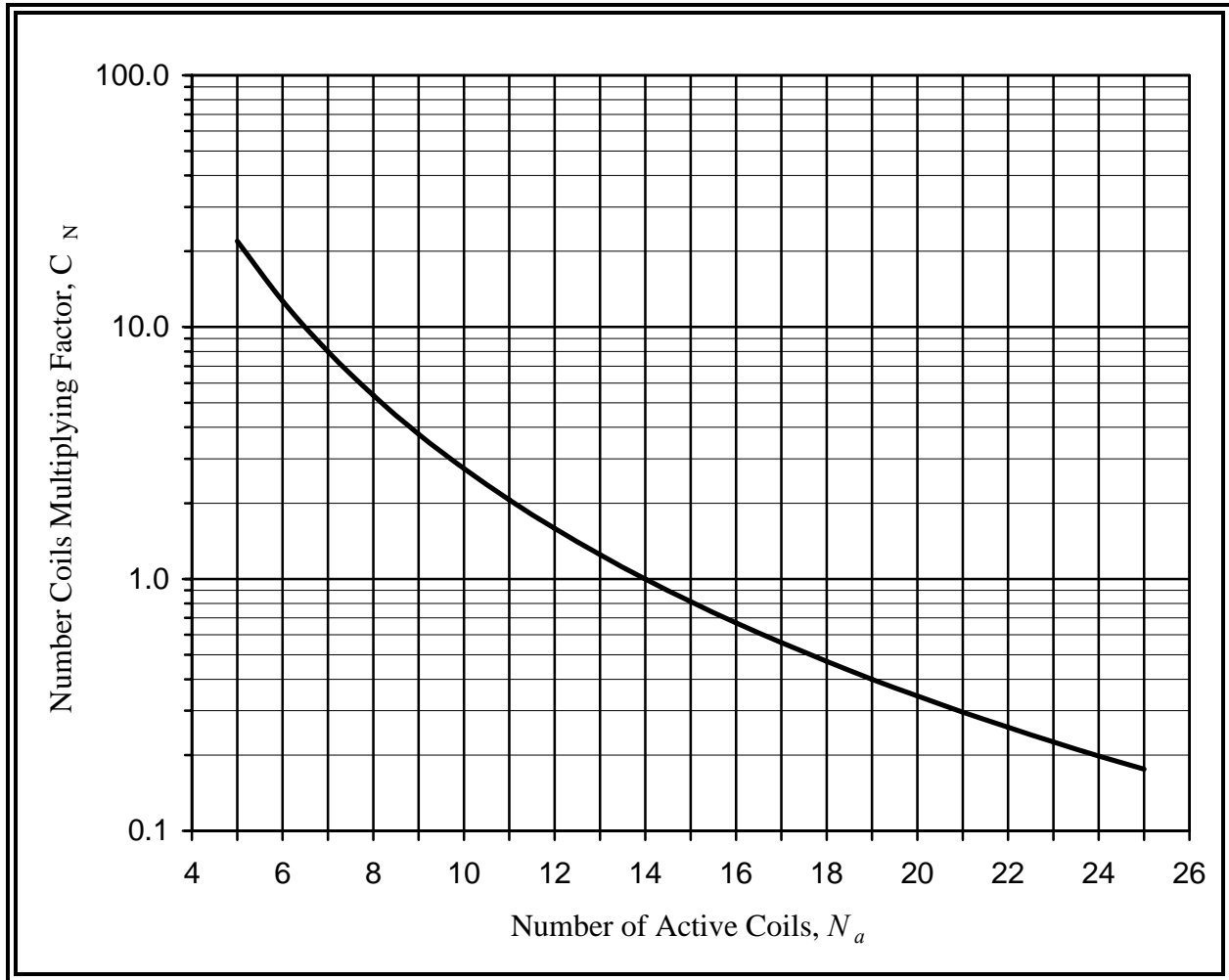
$$C_{DW} = \left(\frac{D_W}{0.085} \right)^3$$

Figure 4.8 Multiplying Factor for Wire Diameter



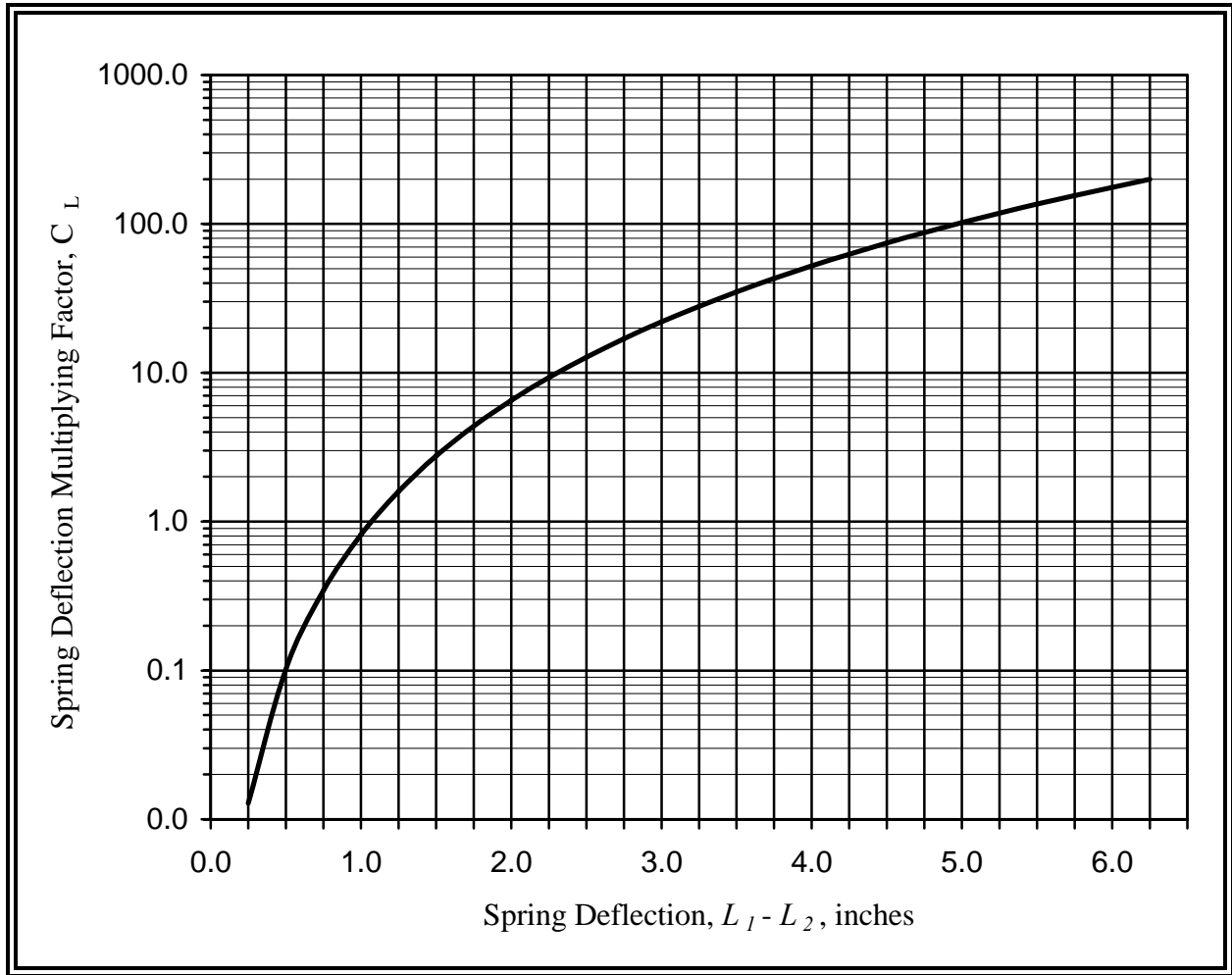
$$C_{DC} = \left(\frac{0.58}{D_C} \right)^6$$

Figure 4.9 Multiplying Factor for Spring Coil Diameter



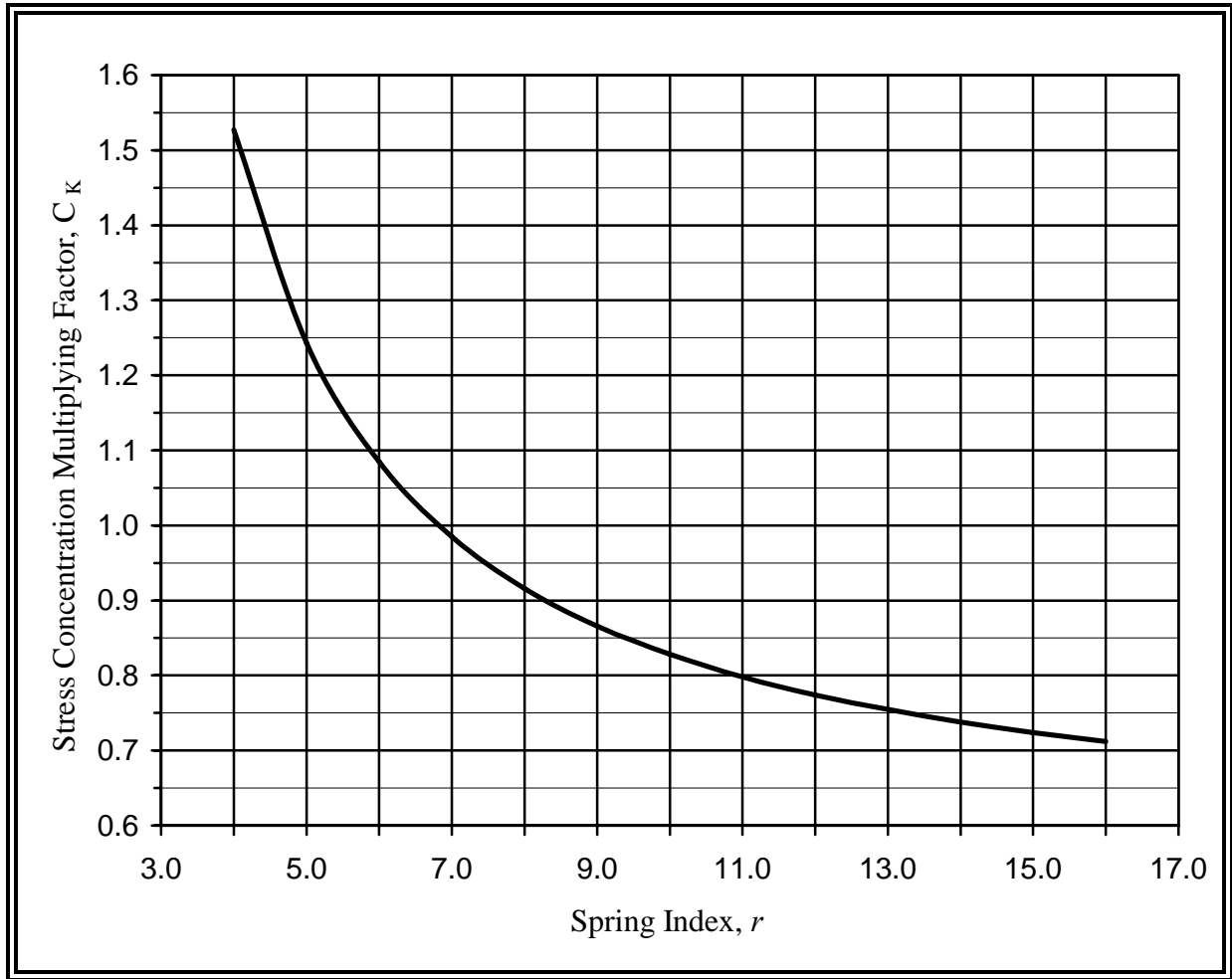
$$C_N = \left(\frac{14}{N_a} \right)^3$$

Figure 4.10 Multiplying Factor for Number of Coils in a Spring



$$C_L = \left(\frac{L_1 - L_2}{1.07} \right)^3$$

Figure 4.11 Multiplying Factor for Spring Deflection



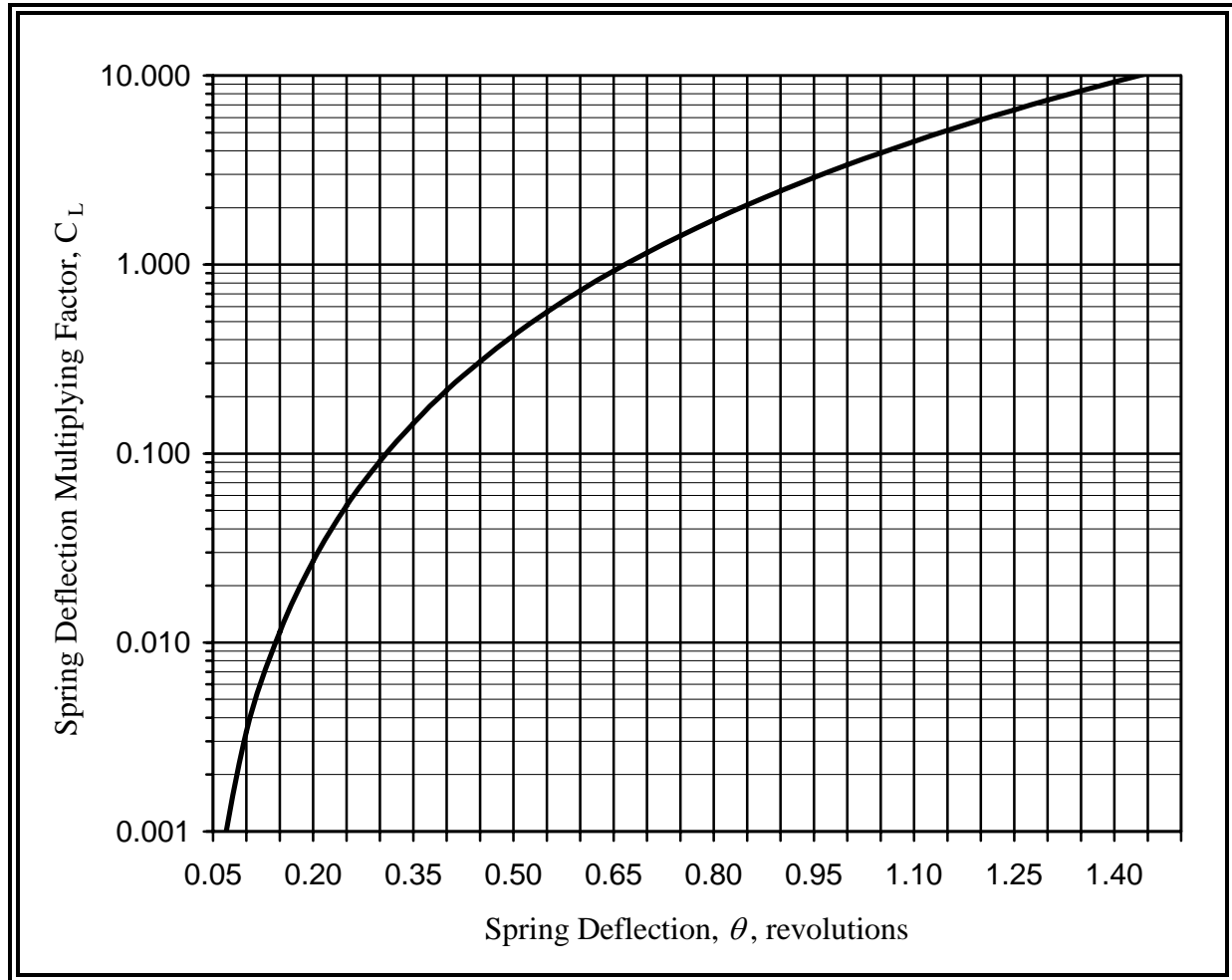
$$C_K = \left(\frac{K_W}{1.219} \right)^3$$

Where: $K_W = \frac{4r-1}{4r-4} + \frac{0.616}{r}$ and $r = \frac{D_C}{D_W}$

D_C = Coil Diameter, inches

D_W = Wire Diameter, inches

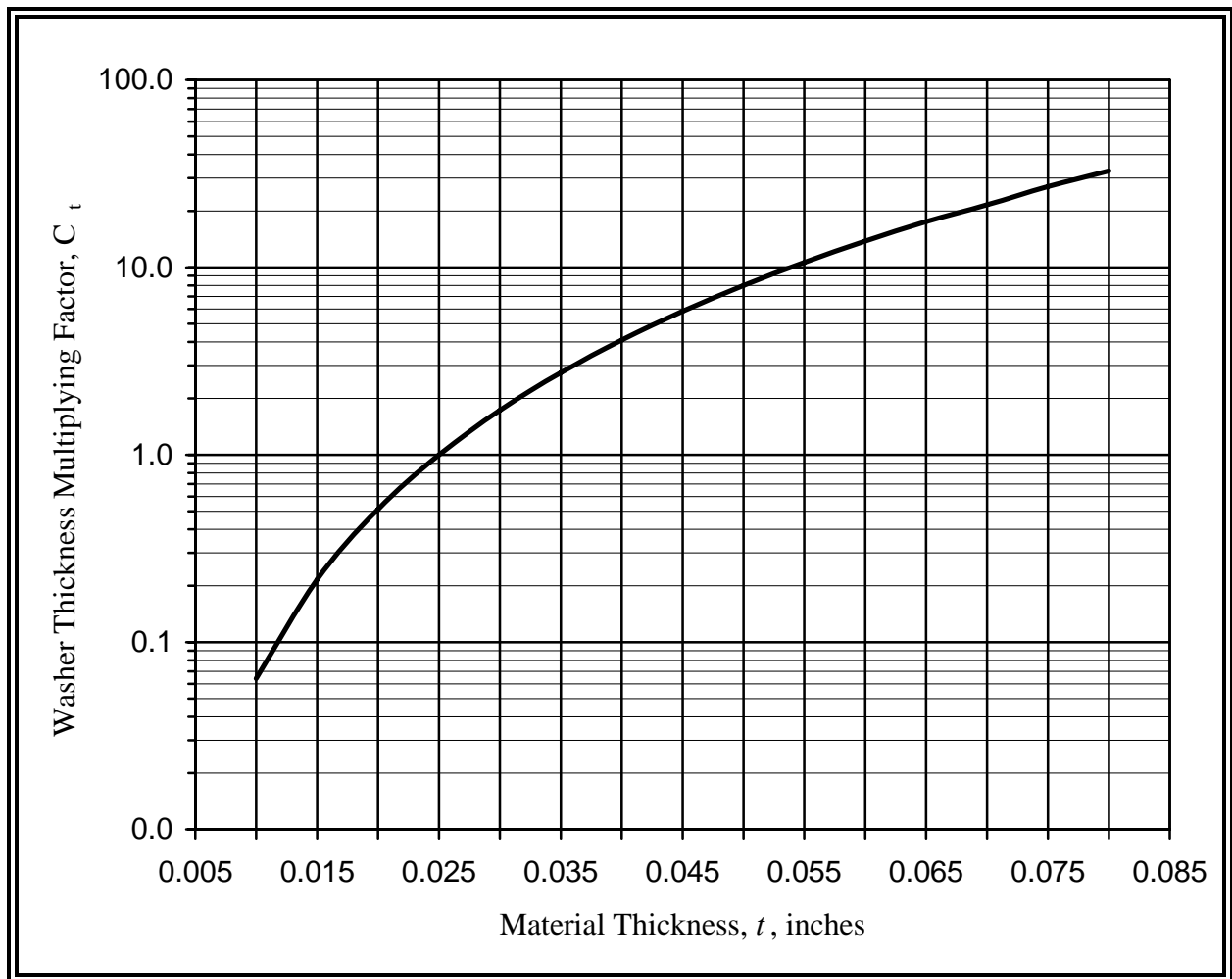
Figure 4.12 Multiplying Factor for Stress Concentration Factor



$$C_L = \left(\frac{\theta}{0.667} \right)^3$$

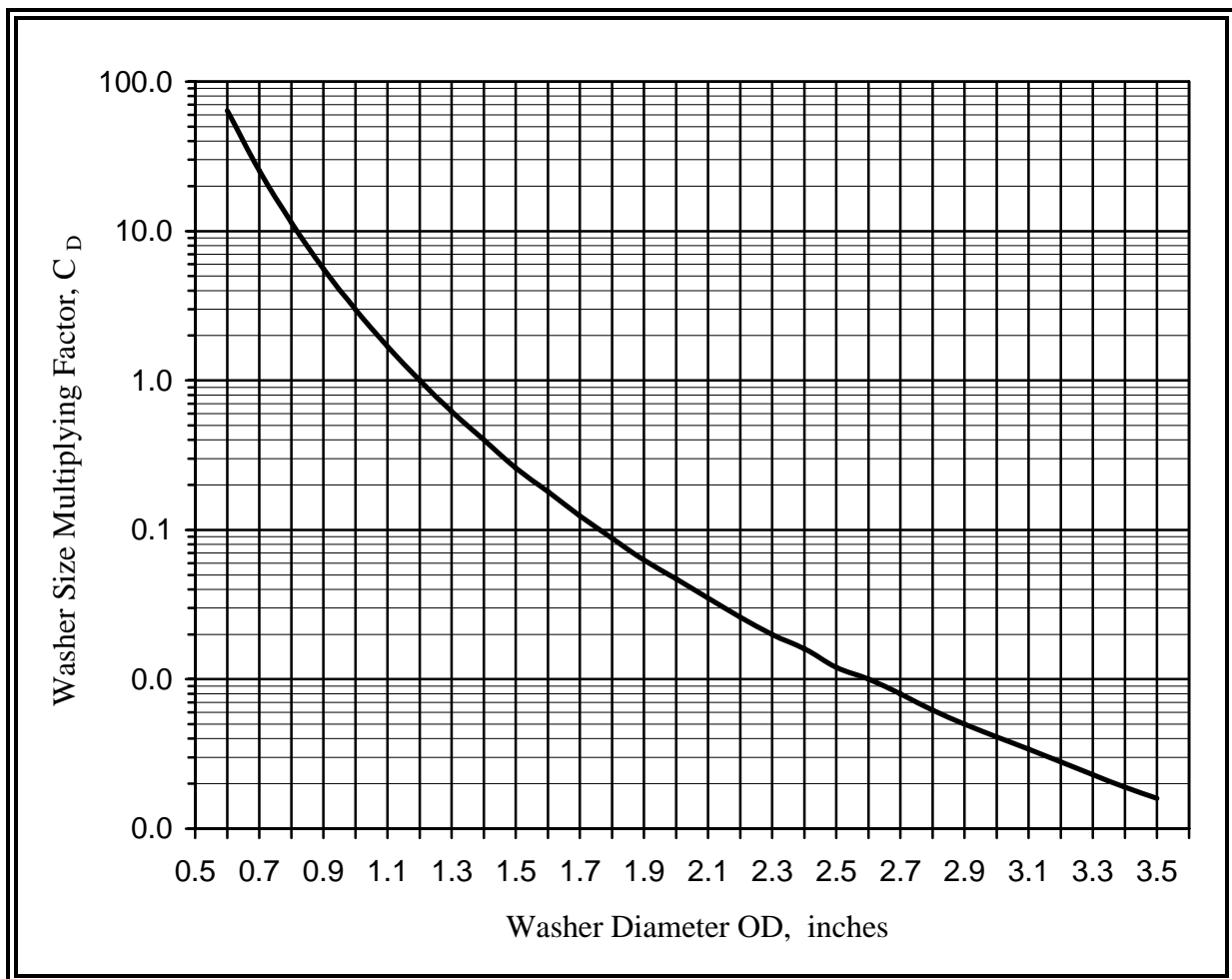
Where: θ = Angular rotation, revolutions

Figure 4.13 Multiplying Factor for Deflection of a Torsion Spring



$$C_t = \left(\frac{t}{0.025} \right)^3$$

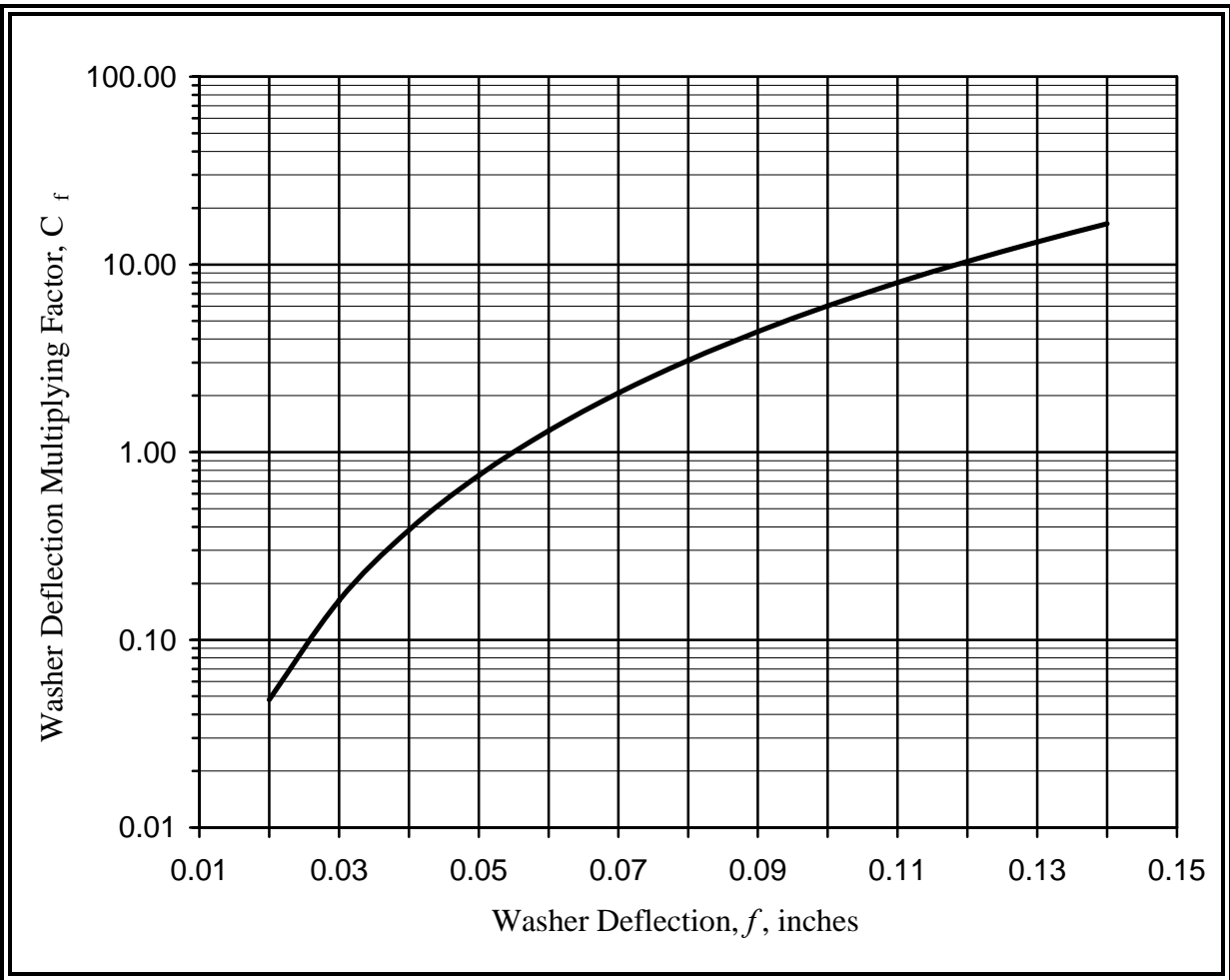
Figure 4.14 Multiplying Factor for Material Thickness



$$C_D = \left(\frac{1.20}{OD} \right)^6$$

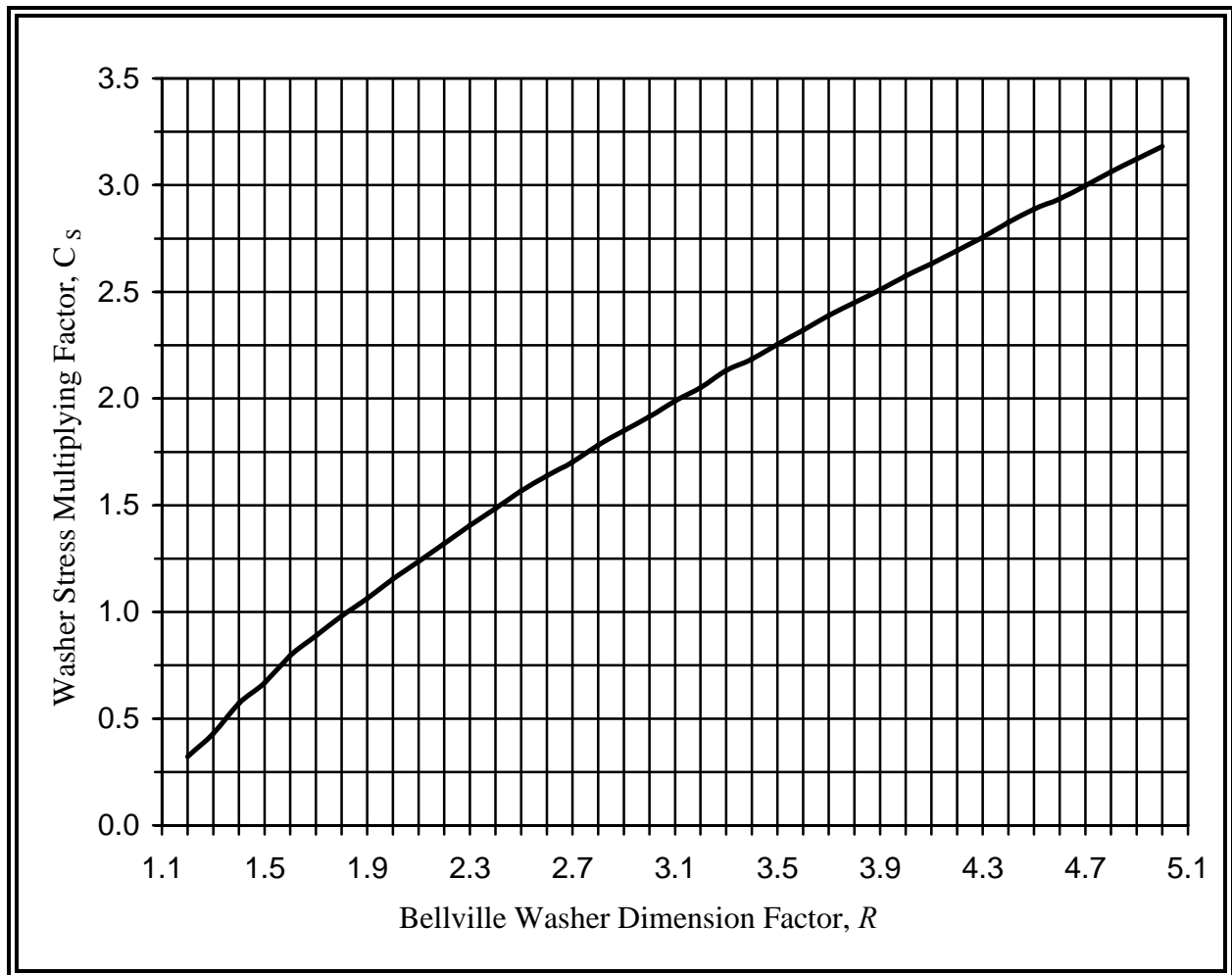
Where: OD = Outside Diameter of Belleville, Curved or Wave Washer, inches

Figure 4.15 Multiplying Factor for Washer Size



$$C_f = \left(\frac{f}{0.055} \right)^3$$

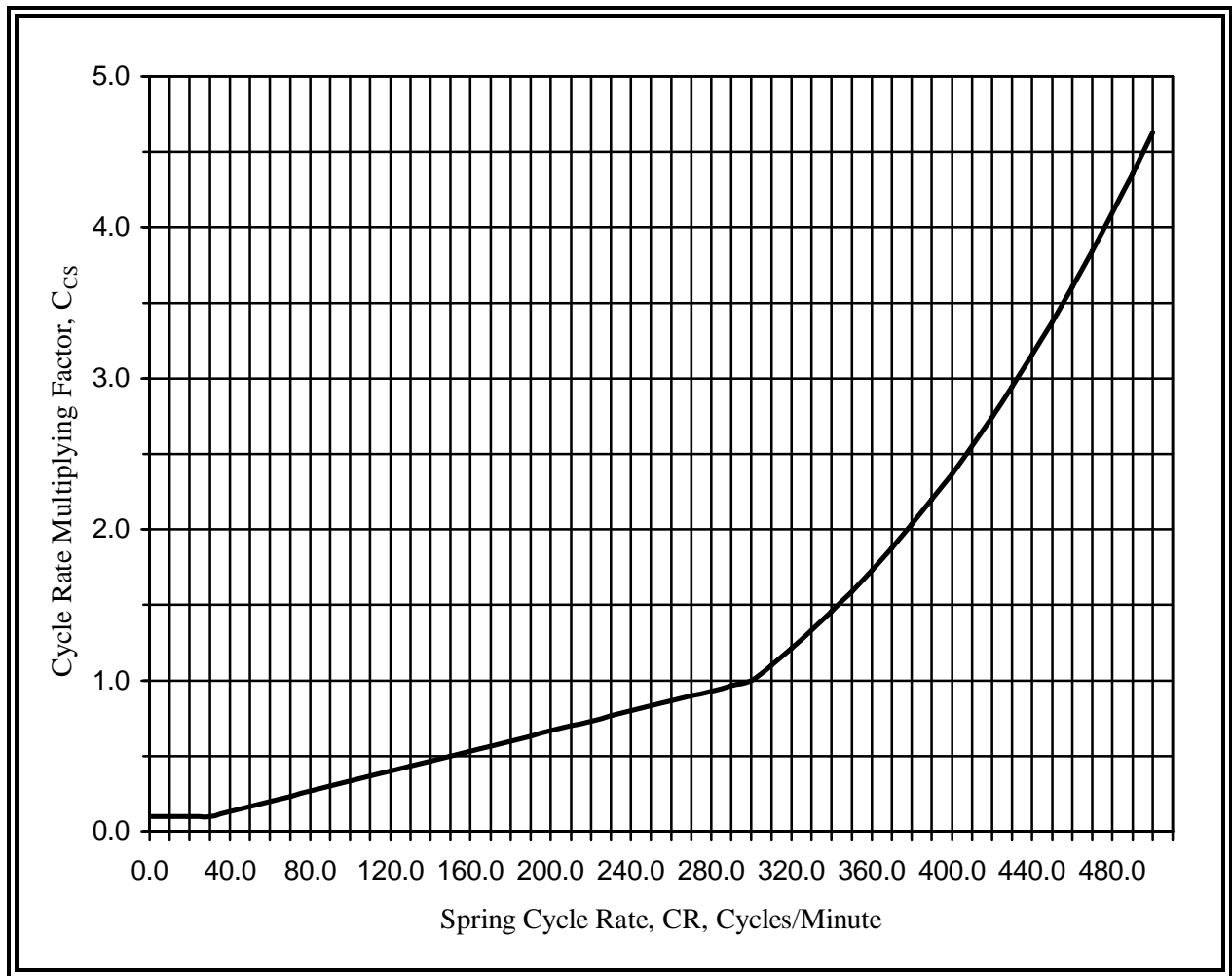
Figure 4.16 Multiplying Factor for Washer Deflection



$$C_s = \left(\frac{6}{\pi \ln R} \right)^3 \left(\frac{R-1}{\ln R} - 1 \right) \left(\frac{R-1}{2} \right) \left(\frac{(R-1)^2}{R^2} \right)$$

Where: $R = \frac{\text{outside diameter}}{\text{inside diameter}}$

Figure 4.17 Multiplying Factor for Belleville Washer Compressive Stress



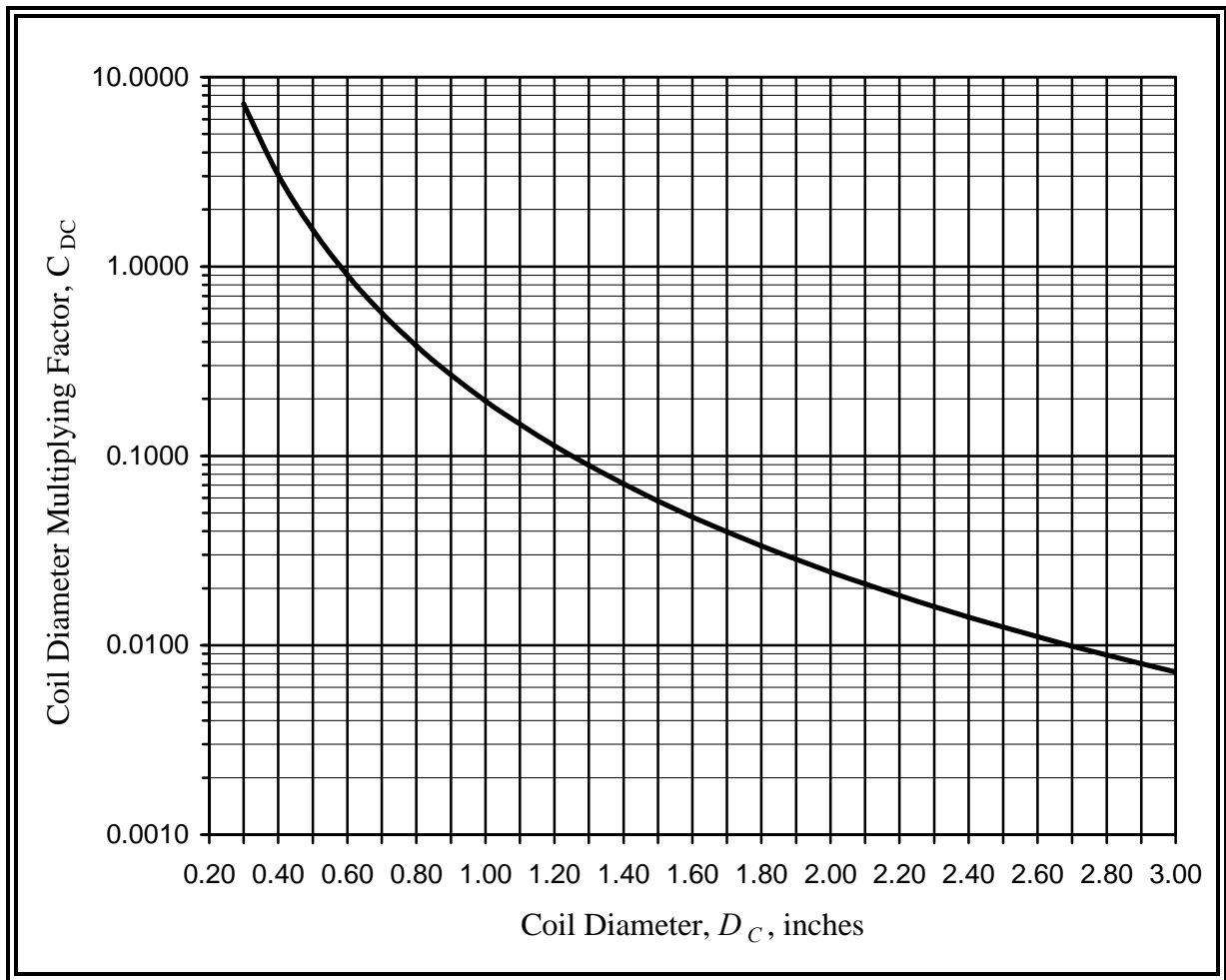
For $CR \leq 30$ cycles/min, $C_{CS} = 0.100$

For $30 \text{ cycles/min} < CR \leq 300 \text{ cycles min}$, $C_{CS} = \frac{CR}{300}$

For $CR > 300 \text{ cycles/min}$, $C_{CS} = \left(\frac{CR}{300}\right)^3$

Where: CR = Spring cycle rate, cycles/min

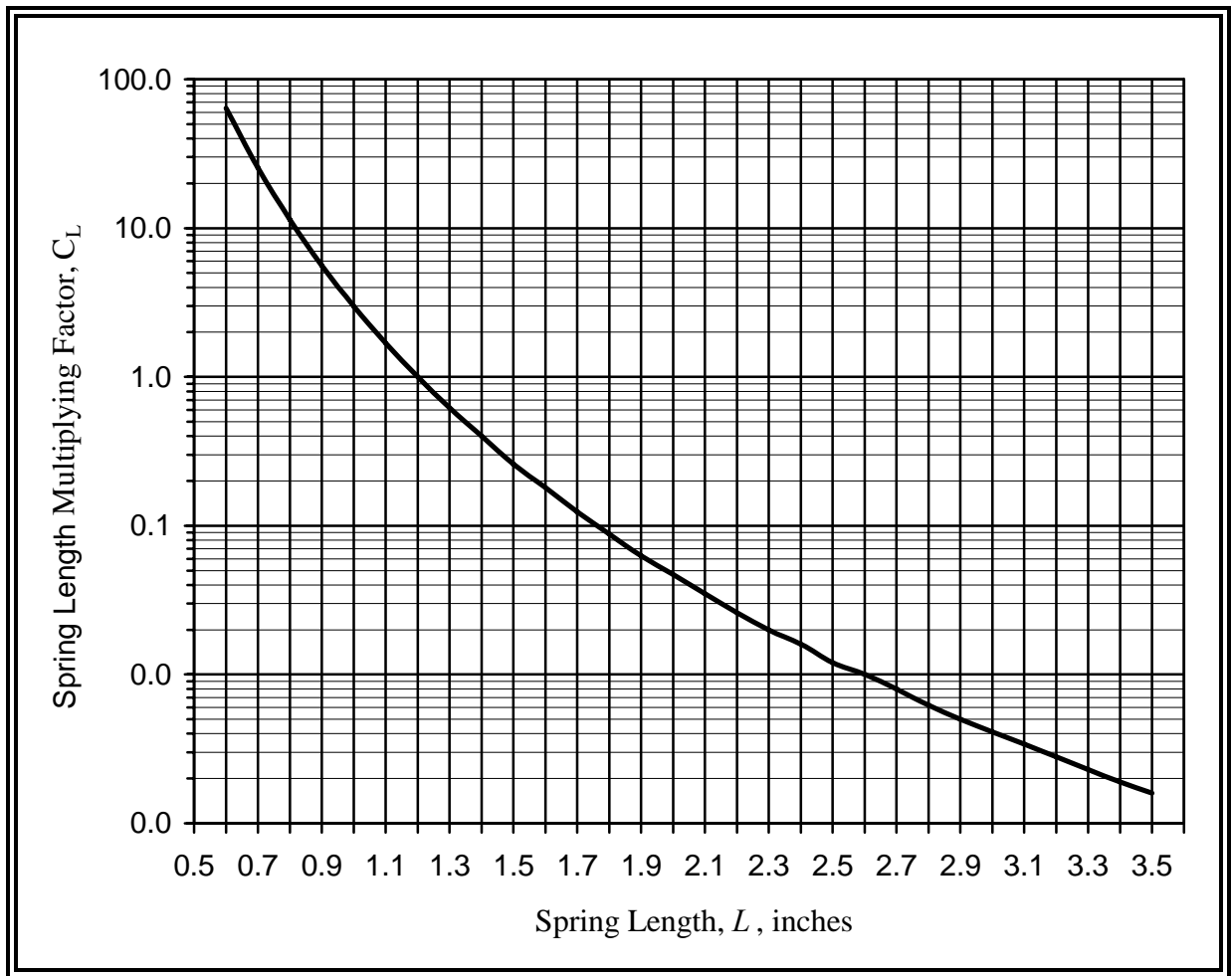
Figure 4.18 Multiplying Factor for Spring Cycle Rate



$$C_{DC} = \left(\frac{0.58}{D_C} \right)^3$$

Where D_C = Coil diameter, inches

**Figure 4.19 Multiplying Factor for Spring Coil Diameter
(Torsion Springs)**



$$C_L = \left(\frac{1.20}{L} \right)^6$$

Figure 4.20 Multiplying Factor for Spring Length

Table 4-2. Moduli of Rigidity and Elasticity for Typical Spring Materials

MATERIAL	MODULUS OF RIGIDITY (G_M) lbs/in² x 10⁶	C_G	MODULUS OF ELASTICITY (E_M) lbs/in² x 10⁶	C_E
Ferrous:				
Music Wire	11.8	1.08	29.0	1.05
Hard Drawn Steel	11.5	1.00	28.5	1.00
Chrome Steel	11.2	0.92	29.0	1.05
Silicon-Manganese	10.8	0.83	29.0	1.05
Stainless, 302, 304, 316	10.0	0.67	28.0	0.98
Stainless 17-7 PH	10.5	0.76	29.5	1.04
Stainless 420	11.0	0.88	29.0	1.05
Stainless 431	11.4	0.97	29.5	1.11
Non-Ferrous:				
Spring Brass	5.0	0.08	15.0	0.15
Phosphor Bronze	6.0	0.14	15.0	0.15
Beryllium Copper	7.0	0.23	17.0	0.21
Inconel	10.5	0.76	31.0	1.09
Monel	9.5	0.56	26.0	0.76

NOTE: Modulus G_M is used for compression and extension springs; modulus E_M is used for torsion springs, flat springs and spring washers.

$$C_G = \left(\frac{G_M}{11.5 \times 10^6} \right)^3 \qquad C_E = \left(\frac{E_M}{28.5 \times 10^6} \right)^3$$

where: G_M = Modulus of Rigidity (lbs/in²)

E_M = Modulus of Elasticity (lbs/in²)

Table 4-3. Material Tensile Strength Multiplying Factor, C_Y

MATERIAL	TENSILE STRENGTH, T_S lbs/in² x 10³	C_Y
Brass	110	5.15
Phosphor Bronze	125	3.51
Monel 400	145	2.25
Inconel 600	158	1.74
Monel K500	175	1.28
Copper-Beryllium	190	1.00
17-7 PH, RH 950	210	0.74
Hard Drawn Steel	216	0.68
Stainless Steel 302, 18-8	227	0.59
Spring Temper Steel	245	0.47
Chrome Silicon	268	0.36
Music Wire	295	0.27

NOTE: These are typical values based on a wire diameter of 0.1 inch. Actual values of tensile strength will vary with wire diameter.

$$C_Y = \left(\frac{190 \times 10^3}{T_S} \right)^3 \quad \text{where } T_S = \text{Tensile Strength, lbs/in}^2$$

Table 4-4. Wave Washer Multiplying Factor, C_{NW}

NUMBER OF WAVES	C_{NW}
3	2.78
4	1.56
5	1.00
6	0.69
7	0.51
8	0.39

$$C_{NW} = \left(\frac{5}{NW} \right)^2$$

4.5 REFERENCES

12. Carson, Harold, "Springs: Troubleshooting and Failure Analysis", Marcel Dekker, Inc. New York. (1983)
14. "Engineering Guide to Spring Design" Associated Spring, Barnes Group Inc., Form No. 515 (1981).
19. Hindhede, U., et al, Machine Design Fundamentals, John Wiley & Sons, NY, 1983
35. "Optimum Design of Helical Springs", Machine Design, (6 November 1980).
58. Parmley, R.O., Mechanical Components Handbook, McGraw-Hill Book Co., NY 1985
82. Metals Handbook, American Society for Metals, 1985, ISBN 0-87170-188-X
115. Partab Jeswani and John Bleda, "A Predictive Process for Spring Failure Rates in automotive Parts Application", General Motors corporation, SAE Technical Paper 910356, February 1991

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